Aircraft routing problem model for fractional fleets using fault prognostics

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Abstract

Purpose – The objective of this work is to provide a novel aircraft allocation model for fractional business aviation. This model may provide decision-makers with alternative routing solutions that take into consideration preventive maintenance and failure prognostics information. The expected results are more efficient routing solutions when compared to conventional planning models, to help decision-makers improve operations and maintenance planning.

Design/methodology/approach – The model is a mixed integer linear problem formulation addressing and considering preventive maintenance and failure prognostics for optimal operations. Numerical experiments were performed using both field and synthetic data to validate the proposed method. All instances are solved using branch, price and cut algorithms from open-source software.

Findings – The results obtained in this study show that the use of failure prognostics information in aircraft routing can provide improvements in overall planning. By choosing slightly longer flight legs, the flight cost will increase, but putting an aircraft with a higher risk of failure on a leg inbound to a maintenance base can reduce maintenance and overall operating cost.

Originality/value – The model and method provide decision-makers with routing solutions that consider new aspects of planning, not used in previous works, such as failure. Most of the literature focuses on solving routing problems for large commercial airlines. Considering that, few solutions are found in literature for fractional business operators, which have their own operational particularities, such as a company managing a fleet of aircraft belonging to multiple shareowners. In such operation, clients may not always fly in the aircraft that they are shareowners, but an aircraft from the fractional fleet of the same category. Here, the company managing the aircraft guarantees that an aircraft will be ready to attend client demands in minimum time. One of the major differences from other models of operation is the dynamic nature of its flight demands, thus requiring flexible and agile planning limiting the available time to find a routing solution.

Keywords Fractional fleet, Failure prognostics, Branch, Price and cut

Paper type Research paper

Acronyms

AMRP	Aircraft maintenance routing problem
ATSP	Asymmetric traveling salesperson problem
B&B	Branch and bound
CAP	Crew assignment problem
CG	Column generation
ICAO	International Civil Aviation Organization
OF	Objective function
TAT	Turnaround time

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TSP	Traveling salesperson problem
US	United States
VRP	Vehicle routing problem

1. Introduction

The aviation sector is divided, primarily, into three segments; military aviation, scheduled airlines and general aviation. General aviation is described as any operation outside of the other two categories. Business aviation is a subdivision of general aviation where aircraft are used for business purposes.

Business operators differ from commercial airlines not only in the size of aircraft operated, but also in the legislation that they must follow. Albeit both are in the major category of civil aviation, commercial airlines are considered scheduled air transport while business aviation is categorized as general aviation in the definition adopted by the International Civil Aviation Organization (ICAO) (ICAOdocument STA/10-WP/7, 2009).

In business aviation, there are a few different models of operations (Yao *et al.*, 2005). The first is the sole owner of an aircraft. In this case, the owner is responsible for all operation and costs. Next is the shared model where partners acquire an aircraft together and establish a proportion of usage depending on the size of the share. However, the owners will still have all the responsibilities as the previous model but with an expected benefit of diluted cost. There is also the leasing model, which is same as the previous ones, the owner is responsible for maintaining and managing the aircraft. This model, however, does not require the lessee to expend the large sum of the price of the aircraft, but pays a periodic fee for its usage. Next, there is the charter model in which the customer acquires the right to use an aircraft for the requested flight and pay usually by the hour. In this way the customer has no additional cost besides those already contracted, but the cost of the flight hour will tend to be significantly higher than the previous models. Finally, there is the fractional model, which is the focus of this work. In this case, the customer buys a share of an aircraft that is managed by a company and that share will give him or her the right to a predetermined number of flight hours per year. These shares can be as small as 1/16th of the aircraft, which in general allows for 50 flight hours per year. In this model, the company is responsible for maintaining and managing the fleet as well as making them available for the client needs. The client will then pay a fixed administration fee and an hourly fee for usage (Hicks *et al.*, 2005; Martin *et al.*, 2003).

This type of operation brings a few unique challenges, among which are the necessity of agile planning due to the short request period and managing conflicting demands from customers.

Since business aircraft tend to be smaller, there is a wider range of airports from which they can operate. This allows greater flexibility in operation as is explained below for a point-to-point network. However, this type of network also poses logistical challenges for the fleet manager, like the lack of a centralized maintenance structure and fewer maintenance resources at smaller airports. The use of these smaller airports increases the flight options from around 400 airports, for scheduled airlines, to more than 5,100 airports, for general aviation, in the USA alone (2019 Annual Report – GAMA, 2018).

As seen in Gronkvist (2005), there are three types of operating networks in the airline industry, illustrated in Figure 1. The first is the linear network, the least used of them all in the aeronautical sector. Here, all the airports are connected by a single tour. In other words, there is a single flight path followed by all aircraft.

Next comes the point-to-point network. For this case, all airports are connected to each other by a single flight. Low-cost commercial airlines and business aviation operators mostly use this type of network, allowing them to bypass busy and expensive airports guaranteeing improved flexibility.

The last type of network and most used by commercial airlines is the hub-and-spoke network. In this type of network, there is a main hub of operation where all flights arrive at or depart from. This model is the preferred method of large commercial airlines because it is easier to mitigate operation disruptions and allows operators to have a single maintenance station to service a large portion of their fleet (Gronkvist, 2005).

For both business and commercial aviation, the operational planning of flights is usually segmented into four main phases (Al-Thani *et al.*, 2016; Basdere and Bilge, 2014; Diaz-Ramirez *et al.*, 2013; Eltoukhy *et al.*, 2017a, b; Khaled *et al.*, 2018; Kohl *et al.*, 2007), shown in Figure 2. The first is the flight scheduling phase, followed by the fleet assignment phase, then the tail assignment or aircraft maintenance routing phase and finally the crew assignment phase.

The flight scheduling phase is when the flight legs are established. This step is crucial for the airliners since only flights with sufficient demand must be established in order to guarantee profitability. However, for fractional operators in the business aviation sector, this

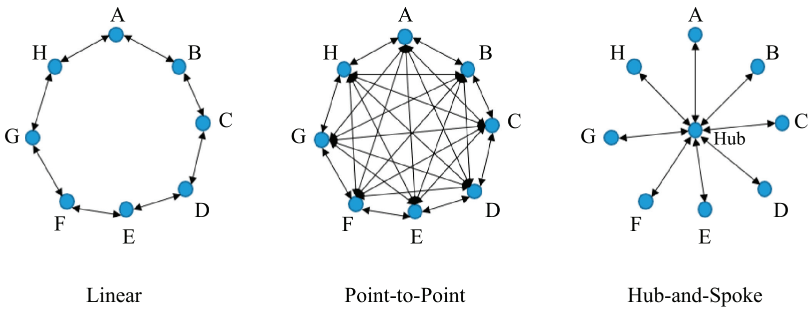

Figure 1.
Illustration of network types

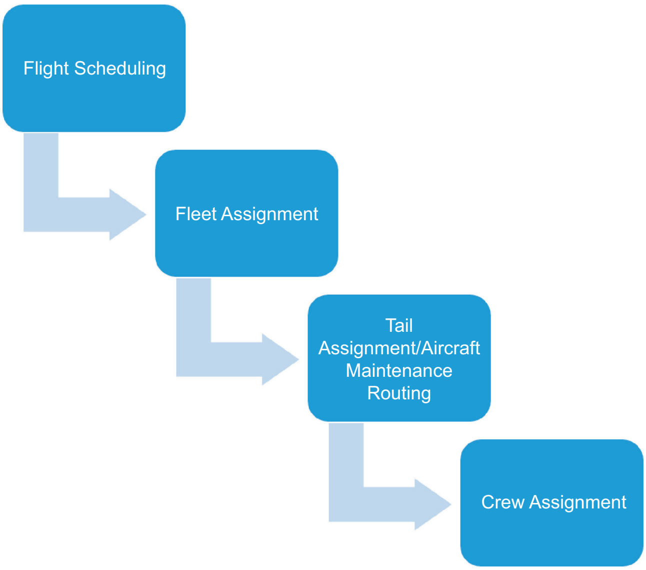

Figure 2.
Operation planning phases

phase does not depend on the operator. The flight demands come directly from the customers and the operator manages the fleet of various owners guaranteeing that all flights are attended.

In the fleet assignment phase, the operator must determine which type of aircraft is more adequate to attend each flight leg. Again, for commercial aviation, this process is also strategic since commercial airlines usually have mixed fleets and each type of aircraft may have a different passenger capacity and flight cost. This implicates the number of seats that will be offered for each flight leg, as well as the fuel consumption, fixed flight costs and eventual operating restrictions due to airport capacity to receive certain aircraft. Fractional owners, on the other hand, own a fraction of a specific aircraft model; hence, the demands coming from a client will be allocated to a fleet of aircraft of the same model that he or she owns. In the case when a model of the aircraft owned by that customer cannot attend the demand, the operator may find an alternative way to attend the customer, including the use of a larger aircraft or paying a chartered flight. Although this aspect of planning is outside of the scope of this work, a heterogeneous fleet is tested to expand the applicability of the proposed model, providing the possibility of merging the fleet assignment and aircraft maintenance routing phases.

The next phase involves building the routes that will connect each flight leg in the demand and allocating a specific tail number to each route. In this phase, commercial airlines have more flexibility in planning due to the hub-and-spoke network. By concentrating maintenance workshops and most passenger flown at hubs, switches between tail numbers are easier, thus mitigating the effects of disruptions. Since fractional operators operate mostly in a point-to-point network, aircraft do not always return to a home base with maintenance capacity and there is no guarantee that flight legs will depart from the same airport where a previous flight arrives. This implies frequent empty flight legs to relocate aircraft in order to attend demands or perform maintenance. This phase will be the focus of this work, since there is a clear opportunity for cost reduction in minimizing empty flight legs.

The last phase deals with pairing crews to aircraft in order to create a roster for employees. This step is also important since the crew wages is one of the highest costs for airlines after fuel. The regulatory aspect of crew labor increases the challenge of this activity, setting strict limitations for working hours and rest periods that may even change from country to country.

Maintenance is a crucial part in flight planning. Apart from being an additional cost in operations, it can also limit its availability. From Ben-Daya *et al.* (2009), we have three main types of maintenance: reactive or corrective maintenance, preventive maintenance and predictive maintenance.

Corrective or reactive maintenance tasks are carried out after a failure occurs. A failure that happens out of a maintenance base incurs in greater costs for the fleet manager since mechanics and equipment may need to be transported to the location of the aircraft.

Preventive maintenance on the other hand is scheduled and has the purpose of restoring the aircraft to a safe state. Preventive maintenance is a usage-based concept in which parts are repaired or replaced at certain intervals, independent of current condition (Samaranayake and Kiridena, 2012). Considering aircraft preventive maintenance, activities can have three types of thresholds: flight hour limits, calendar limits and cycle limits (Martin *et al.*, 2003).

Predictive maintenance is a concept where cost-effective tools are used to monitor the condition of critical equipment. Instead of relying on average life statistics, direct monitoring estimates the remaining useful life (RUL) of the part (Mobley, 2002).

Failure prognostics information is a potentially powerful tool in improving aircraft routing. By allocating aircraft with higher failure probabilities to flight legs inbound for maintenance bases, the maintenance costs can be reduced, as the aircraft will be somewhere prepared for aircraft-on-ground (AOG) events. In this study, AOG events are considered the faulty situations when an aircraft is not able to be dispatched unless the failure is repaired.

This information may also help operators to engage troubleshooting resources such as spare parts and other support equipment before the event occurs, reducing lead times and costs. Since fractional operators have multiple maintenance bases and a large amount of possible flight routes, being able to plan for corrective maintenance events could give fractional operators a competitive advantage in terms of fleet availability and maintenance costs.

The main objective of this work is to formulate mathematically and solve the routing problem for fractional business operations including preventive maintenance and failure prognostics aspects and data.

Within the context of the problem, this work contributes to the exploration of an important and promising part of the problem as a whole. The modeling, even with the premises observed, addresses the supportability of homogeneous and heterogeneous fleets of executive aircraft in a fractional fleet when scheduling flights taking into account both legacy standards of preventive maintenance and the use of information from prognostic models. Naturally, this work paves the way for other modeling by eliminating the premises and completing the systemic understanding of the aircraft allocation problem.

This text is organized in the following manner. Section 2 presents a review of the literature concerning the topics of this work. The methodological approach is detailed in Section 3, and the application of the method and discussion are presented in Section 4. Finally, Section 5 concludes the research.

2. Literature review

In the literature much attention has been focused on large commercial airline problems, (Eltoukhy *et al.*, 2017a, b; Haouari *et al.*, 2011; Khaled *et al.*, 2018; Kohl *et al.*, 2007; Liang and Chaovalitwongse, 2012; Liang *et al.*, 2015; Maher *et al.*, 2018; Warburg *et al.*, 2008). On the other hand, few researches concerning the business aviation operation have been found. Some of these works include Martin *et al.* (2003), Yao *et al.* (2005) and Yao *et al.* (2008).

In works focused on fractional operations, the main aspect treated was the crew-scheduling problem. Martin *et al.* (2003), Hicks *et al.* (2005), Yao *et al.* (2005) and Yao *et al.* (2008) all solved the crew-scheduling problem using aircraft maintenance routing as a feasibility constraint, thus not necessarily optimizing the maintenance routing.

As explained in Eltoukhy *et al.* (2017a) and Maher *et al.* (2018), there are many models used to construct a routing solution for aircraft. Some authors used a string-based approach where the strings are a sequence of connected flights. Generally, for airlines, the strings begin and end at the same base. This method is usually formulated as a set partitioning problems and solved using a branch and price strategy. This method has one drawback, which is the large number of strings generated taking a large computational time.

Another approach is the network-based method. This method can be solved in a considerably smaller amount of time when compared to the string-based method. The network model uses timelines for different stations, including airports and maintenance stations, in order to depict the flow of the aircraft as shown in Liang and Chaovalitwongse (2012).

The third method of solution is the big cycle approach. Some authors associated the aircraft routing problem with the asymmetric traveling salesperson problem (ATSP) due to the similarities between them (Clarke *et al.*, 1997; Mak and Boland, 2000). The first focuses on finding feasible maintenance rotation problem by formulating the problem of aircraft maintenance routing problem (AMRP) as an ATSP and the second solving with metaheuristics the AMRP formulated as an ATSP.

In commercial aviation, the planning horizon is a well-defined parameter when it comes to establishing operations. In this way, the planning can be done in such a way to create cyclical routes that repeat in periods of days, weeks or even months. By doing this, the distribution of activities among aircraft can be more easily controlled. In commercial aviation, this is crucial due to the large amount of flights and aircraft to manage.

However, for the business aviation operator, the planning horizon is in most part out of the hands of the operator. Since the flights are determined by customer demand, the request can come from months in advance to as soon as a few hours from departure. This stochastic demand complicates planning to the point that maintenance activities may overlap flights and cause unforeseen unavailability.

The main works dealing with fractional ownership problem are presented in Table 1. Martin *et al.* (2003) present an integrated system that provides routing solutions based on a mixed-integer linear programming model solved with CPLEX. The focus of this work is primarily in crew scheduling and some simplifications are used to merge aircraft and crew scheduling and preventive maintenance is treated as fixed stops in the planning. Hicks *et al.* (2005) focused on modeling constraints and cost factors for another integrated system used for fractional operations. To solve this model, they used GENCOL, a column generation based software developed by GERAD, an operations research center. This formulation allows 15-min delays in planning. Yao *et al.* (2005) continue to study the effects of flexible time windows for departure times, which was solved using CPLEX. This approach showed promising results when compared to heuristically determined routes used by the fractional operator. By using more flexible crew swapping strategies, departure times and modifying demand, Yao *et al.* (2008) are able to improve operating costs in their study. A column generation approach is used in this study and maintenance events are included in the planning as they occur and the problem is resolved after that. Finally, Munari and Alvarez (2019) continue to use flexible time windows for flight departures, anticipating or delaying flights, in planning. The main contribution in this work is allowing clients to be upgraded to a larger aircraft if the upgrade will result in a lower overall cost. Flight upgrades usually happen when no aircraft of the contracted category are available for a client, in this case, an aircraft of a higher category is made available even if its operating cost is higher.

Table 1 provides a comparison between the major works treating business aviation cases, including fractional operations. Here the solution methods are compared as well as the overall contributions.

2.1 Maintenance

The models in previous literature introduce maintenance requirements in various ways. Most works create mandatory flight legs in the demanded activities, which have the duration of the checks, and the origin and destination of the "flight" are the same maintenance station.

Table 1. Literature review comparison

Authors	Solution method	Contribution
Martin *et al.* (2003)	CPLEX	Development of a decision support tool
Hicks *et al.* (2005)	CG (GENCOL)	Constraint and cost factor modelling
Yao *et al.* (2005)	Set partitioning, CPLEX	Flexible time windows for departure time
Yao *et al.* (2008)	Set partitioning, CG	Considering demands from customers that are not fractional owners
Munari and Alvarez (2019)	CPLEX	Inclusion of service upgrade possibility
This work	B, C and P (Gurobi)	Failure prognostics considered during routing

Note(s): CG = Column Generation, B, C and P = Branch, price and cut

In this work, however, we treat these maintenance events with more flexibility, by separating the calendar-specific and flight-hour-specific maintenance activities and allowing a wider window to accomplish them.

Despite many works treating the problem of including maintenance activities in the planning process, few acknowledge the different types of preventive maintenance (Khaled *et al.*, 2017; Khaled *et al.*, 2018; Martin *et al.*, 2003) and the possibility of having flexibility in maintenance allocation (Munari and Alvarez, 2019).

2.2 Failure prognostics

IVHM is an integrated view of a system of systems, monitoring the health of each system to assist in the decision-making process (Jennions, 2013). In this way, this approach provides the ability to recognize, evaluate, isolate and mitigate faults in the system (Jiang *et al.*, 2017). An important part of IVHM is the prognostics and health management (PHM).

PHM provides an estimated RUL for components or systems based on collected data and estimated future usage. Modern aircraft provide more data than older models and thus an opportunity to improve operations and maintenance planning.

Over the last years, many works used PHM and IVHM to improve maintenance, design and thus reduce operating costs. Vianna *et al.* (2015) used PHM to estimate AOG events and better plan aircraft line maintenance. Scanff *et al.* (2007) researched the impact of PHM on life cycle cost (LCC) for helicopter avionics. The RUL provided by PHM is also used in a system-level analysis to aid in the maintenance decision process regarding component replacement (Rodrigues *et al.*, 2015).

Rodrigues *et al.* (2012) study possible opportunities brought on by using PHM techniques for aircraft operators. The main aspects touched in this work are inventory management optimization, scheduled maintenance planning, reduction of unscheduled maintenance tasks, improved troubleshooting and intelligent aircraft allocation. The latter is the area of interest of this work. All of these topics also have potential benefits for personnel management, helping to isolate failure causes more rapidly and efficiently and planning appropriate man power for each maintenance event beforehand. The two most prominent benefits cited are increased fleet availability and reduced operational costs, by placing technicians and parts closer to predicted maintenance events, reducing logistics costs and mean time to repair.

In this work, we use a small part of PHM, in the form of failure prognostics, in an attempt to improve aircraft routing and reduce maintenance costs. Usually, the most sought after information derived from failure prognostics is the RUL of an equipment or system. Due to the uncertain nature of this type of prediction data, there are some inherent variabilities in RUL. The two main uncertainties are the actual failure threshold and the evolution of the degradation (Tobon-Mejia *et al.*, 2012). Therefore, the RUL is better represented as a distribution rather than a certain value.

2.3 Branch, cut and price

Branch and bound algorithms are designed to solve discrete and combinatorial optimization problems (Ralphs *et al.*, 2010). It consists of enumerating candidate solutions for a relaxed version of the original problem, allowing infeasible solutions. At each branch, the solutions with variables that do not obey certain restrictions, such as integrality restrictions, are branched out further until feasible solutions are obtained, thus forming a tree of solutions. Each branch is compared to upper and lower branches in order to obtain the best solution.

The efficiency of the branch and bound method depends on the tightness of the relaxation applied. In this context, the branch and cut algorithms provide a relaxed set of solutions that are closer to the feasible solution set than the simpler branch and bound strategy. By adding globally valid inequalities, the search space is reduced as they affect all branches of the solution tree (Ralphs *et al.*, 2010). One of the ways to generate these inequalities is to use Dantzig–Wolfe decomposition (Desrosiers and Lübbecke, 2010).

Branch and price is another method used to tighten the relaxation of the original problem. Here, this is done by column generation. This may result in a large amount of variables in the formulation, therefore the initial relaxation starts with a small subset of variables (Ralphs *et al.*, 2010; Desrosiers and Lübbecke, 2010).

The combination of the two previous methods results in the branch, price and cut algorithm. By using both strategies, the solution space is established more efficiently.

3. Methodology and modeling

The modeling and numerical implementation used in this work were applied to two cases, the first, a standard planning approach without considering the available failure prognostics and the second, a planning considering failure prognostics.

In conventional planning, routing is defined in such way that connection costs are minimized and preventive maintenance is performed accordingly. In the proposed model, also failure prognostics are considered, whereas connection costs are weighed against the maintenance cost considering aircraft that present a higher failure probability prioritizing flight legs inbound for maintenance bases.

Each flight leg of these sequences has an origin, a destination, a departure time and a duration. Turnaround time (TAT) after landing is assumed constant, and the flight time between the cities involved is also known. At the initial planning period, the position of each aircraft is known, as well as its accumulated flight hours.

For this work, the individual cost of operating flights with each aircraft is considered the same (no aircraft is more efficient than any other), the factor that will most influence in planning is the connection between each flight.

Ideally, there would be no connection flights in the final planning, as is the case for most commercial airline operations. This is not, however, a realistic assumption for fractional operators. Thus, minimizing the flight time necessary in connection flights is crucial to obtaining better solutions.

Since it is not the focus of this work to generate failure prognostics data, the failure prognostics used is obtained from supervised machine learning algorithms that use message history from the central maintenance computer and failure occurrences from maintenance and pilot reports. The method was based on the solution presented in (Baptista *et al.*, 2016). The failure prognostics information used in this work is considered as a discrete failure probability distribution as illustrated in Figure 3.

This expected RUL changes with the rolling time windows used in this work, simulating an evolution of the failure mode being monitored. The important aspect of this data for this work is the form of the failure prognostics information that is fed to the proposed model.

Some assumptions are made in order to treat the problem and are presented further:

(1) The position and schedule of each aircraft are known at the moment of planning.

(2) Customer requests can be accepted as soon as 6 h prior to departure.

(3) The flight time between each city is considered constant and calculated as an average time of known flights plus an SD, following a normal distribution.

(4) After a constant TAT, the aircraft will be ready for take-off.

(5) The inability to attend a flight will incur a cancellation fee for the operator.

(6) Connection between every city is permitted.

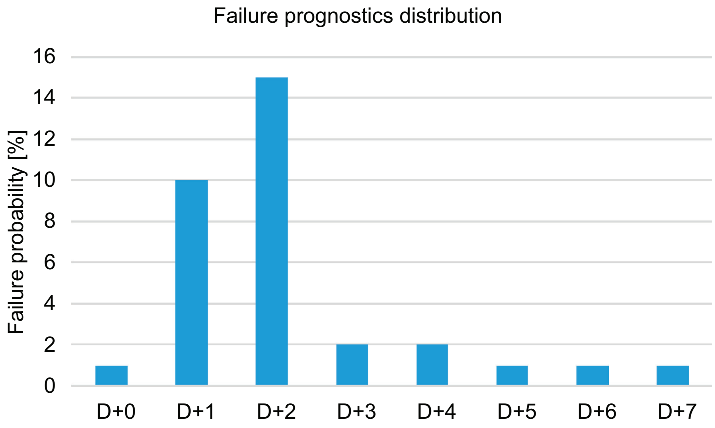

Figure 3.
Failure prognostics distribution

(7) Only one aircraft can attend each flight.
(8) The fleet is composed of only one type of aircraft.
(9) Only one failure occurs at a time.
(10) Maintenance activities are performed in sequence.
(11) Monitored failure result in AOG events.
(12) Penalties are constant for maintenance events and cancelled flights.

Figure 4 depicts a small scenario of how the problem is formulated in the present work. In this case the demands are denoted as i, j and k, each with its departure and arrival locations, departure time s and duration dur. Each possible connection between activities is represented by x and has duration c. As not to pollute the image and have a clearer idea of operations,

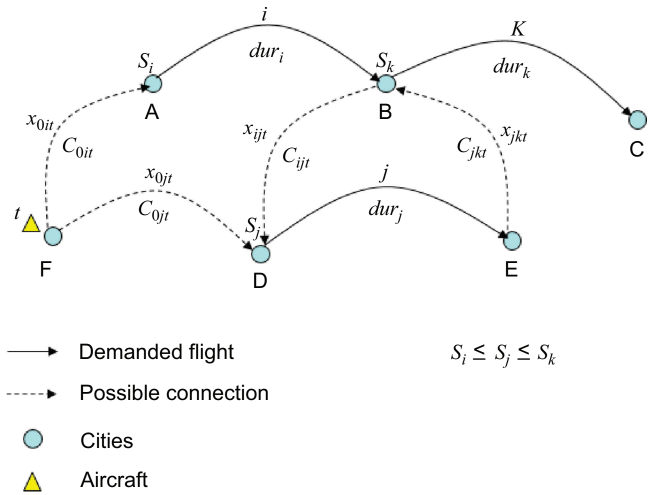

Figure 4.
Depiction of the formulation used in this work

some of the connection arcs are omitted in Figure 4. These arcs are x_{0kt}, x_{ikt}, x_{jit}, x_{kit} and x_{kjt}. In this case, the connection between i and k, x_{ikt} is not explicit in the figure, but would present a duration of $c_{ikt} = 0$ since flight k departs from the location where flight i arrives, assuming that $s_k \geq s_i + \mathrm{dur}_i + \mathrm{TAT}$.

The mathematical formulation developed in this work is described further.

Objective function:

$$\min \sum_{t\in T}\sum_{i\in F'}\sum_{j\in F}(C_{\mathrm{FH}}(c_{ijt} + \mathrm{dur}_j)x_{ijt}) + N_c\ C_c\ -\ \sum_{t\in T_P}\sum_{i\in F'}\sum_{j\in P}(f\ \ x_{ijt}) \tag{1}$$

Subject to:

$$\sum_{t\in T}\sum_{i\in F'} x_{ijt} = 1, \quad \forall j \in F \tag{2}$$

$$\sum_{j\in F} x_{ijt} - \sum_{j\in F} x_{jit} \geq 0, \quad \forall i \in F', \ \forall t \in T \tag{3}$$

$$\sum_{j\in F} x_{ijt} \leq 1, \quad i = 0, \forall t \in T \tag{4}$$

$$N_c + \sum_{t\in T}\sum_{i\in F'}\sum_{j\in F} x_{ijt} = N_{tot} \tag{5}$$

$$\sum_{j\in M} x_{ijt} \leq 1, \quad \forall i \in F', \ \forall t \in T \tag{6}$$

$$s_{it} + c_{ijt} + \mathrm{dur}_i + K_{ijt}(x_{ijt} - 1) - s_{jt} \leq 0, \ \forall i \in A', \ \forall j \in A, \ \forall t \in T \tag{7}$$

$$a_j \sum_{i\in A'} x_{ijt} \leq s_{jt}, \quad \forall j \in A, \ \forall t \in T \tag{8}$$

$$b_j \sum_{j\in F} x_{ijt} \geq s_{jt}, \quad \forall j \in A, \ \forall t \in T \tag{9}$$

$$s_{0t} = 0, \quad \forall t \in T \tag{10}$$

$$y_{it} + c_{ijt} + \mathrm{dur}_i + K2_{ijt}(x_{ijt} - 1) - y_{jt} \leq 0, \ \forall i \in A', \ \forall j \in A, \ \forall t \in T \tag{11}$$

$$\lim_t \sum_{i\in A} x_{ijt} \geq y_{jt}, \quad \forall j \in A, \ \forall t \in T \tag{12}$$

$$y_{0t} = \mathrm{FH}_t, \quad \forall t \in T \tag{13}$$

$$x_{ijt} \in \{0, 1\}, \quad \forall i \in F', \ \forall j \in F, \ \forall t \in T \tag{14}$$

Where T is the set of aircraft and T_P is a subset of T containing the aircraft that have a probability of failing as given by the failure prognostics method. F is the set of flights and F' is the set of flights including the origin of each aircraft. P is a subset of F containing flights whose destination coincides with a maintenance base such that $P \subset F$. The connections to maintenance bases are given in set M. A and A' represent the union of M with F and F', respectively.

Equation (1) defines the objective function that aims to minimize the total costs of connections between flights and cancelled flights. In the formulation, x_{ijt} is a binary variable equal to 1 if aircraft t operates flight i followed by flight j and zero otherwise. The flight hours to connect from flight i to flight j and the duration of flight j are represented by c_{ijt} and dur_j, respectively. The number of cancelled flights is determined by the variable N_c. C_{FH} is the cost per flight hour and C_c is the cost for a cancelled flight. One of the points that set the proposed model apart from previous works is the third part of the objective function, here we establish a factor f that encourages the use of priority flights by aircraft with a risk of a failure occurrence. The priority flight legs and higher risk tail numbers are obtained from the failure prognostics information.

In this sense, when a failure probability is identified by a prognostics system, the model reads that information and applies a cost reduction factor to each priority flight leg whose departure date precedes the most likely date of the failure occurrence as per the failure prognostics distribution. This cost reduction factor is then reduced as the departure dates get further and further from the expected failure date.

Equation (2) denotes constraints that guarantee that each flight is flown once and by only one aircraft. The continuity constraints defined by Equation (3) ensure that an aircraft connecting to a given flight also connects from it and Equation (4) makes it so that each aircraft departs from its position at the beginning of the planning period. Equation (5) verifies the number of cancelled flights by verifying that the sum of cancelled flights, N_c, and operated flights, x_{ijt}, are equal to the total number of flights, N_{tot}.

To establish that each maintenance activity will only be performed once and at a single maintenance base, we have Equation (6). Equation (7) sets the time windows for each flight while constraints (8) and (9) set the respective lower and upper bounds of the time windows. The initial time windows are fixed by Equation (10). In these equations s_{it} and s_{jt} are the time windows at which aircraft t starts flight i and j, respectively. The duration of flight i is expressed as dur_i. K_{ijt} is a large enough number such that constraint (7) is deactivated when $x_{ijt} = 0$ and is given by $K_{ijt} = c_{ijt} + \mathrm{dur}_i + a_i + s_i$.

Similar to constraint (7), constraint (11) determines the flight hours accumulated by each aircraft given the allocated flights. Here, y_{it} indicates the previous accumulated flight hours since completing flight i and y_{jt} the accumulated flight hours after completing flight j for aircraft t. Analogous to K_{ijt}, $K2_{ijt} = y_{it} + c_{ijt} + \mathrm{dur}_i + \mathrm{lim}_t$. Constraints (12) prohibit the accumulated flight hours from exceeding the utilization limit of each aircraft, where lim_t is the maximum flight hours each aircraft t is allowed to fly before needing preventive maintenance. The accumulated flight hours of each aircraft at the start of planning are set in constraints (13), FH_t is the known accumulated flight hours of each aircraft t.

Constraints (7)–(13) also set this model apart from previous works as they distinguish calendar and flight hour based preventive maintenance giving each a distinct window to be accomplished based on aircraft usage.

Equations (14) set the variables x_{ijt} as binary.

The total cost analyzed in this study is composed of the flight hour costs from connections and flight durations, cancellation costs, preventive maintenance costs and corrective maintenance costs as described in Equation (15). The total maintenance cost is the sum of corrective and preventive maintenance costs, as shown in Equation (16). Here, C_{pm} is the preventive maintenance cost and C_{cm} is the corrective maintenance cost.

$$\sum_{t\in T}\sum_{i\in F'}\sum_{j\in F}(C_{\mathrm{FH}}(c_{ijt} + \mathrm{dur}_j)x_{ijt}) + N_c\ C_c + \sum_{t\in T}\sum_{i\in F'}\sum_{j\in M}(C_{pm}x_{ijt}) + C_{cm} \tag{15}$$

$$\sum_{t\in T}\sum_{i\in F'}\sum_{j\in M}(C_{pm}x_{ijt}) + C_{cm} \tag{16}$$

A small alteration is introduced into the objective function and cost equation, Equations (1) and (15), in order to support a heterogeneous fleet of aircraft. Because each aircraft is already treated as a unique individual in this routing formulation, by making the flight hour cost variable, C_{FH}, dependent of aircraft t, C_{FH_t}, a different flight hour cost can be assigned to each aircraft. Although the average flight hours between different bases were not altered in this work for different types of aircraft, no alterations to the presented formulation are needed to apply this, only the input data of c_{ijt} would need changing.

Two different scenarios are considered in this work; one considering the traditional planning without considering any prognostics data and the other where prognostics data is taken into consideration during the planning.

In the first scenario, a standard solution is obtained from a formulation that does not take failure prognostics data into account. The second scenario on the other hand generates a solution that considers available failure prognostics in an attempt to avoid failure occurrences out of base.

4. Results and discussion

All tests presented in this work were implemented and run in RStudio version 1.0.143 using an Intel Xeon 2.70 GHz desktop with 64 GB RAM, running Windows 7 Professional 64 bit operating system. Gurobi optimizer 8.1.1 was used to solve the instances in this study. The objective function and all restrictions were built into a Gurobi model structure, including vectors containing the objective function, inequality signs, constraint limits, variable types and a constraint matrix. No execution time or iteration limits were set, and default parameters were used.

Each data set is composed of 117 flights, for the real case, and 119 flights, for the generated cases, 10 aircraft and 2 available maintenance bases. The data sets have 95 bases where aircraft can go to, including the two maintenance bases. Homogeneous and heterogeneous fleets are also considered in this work. For this, the ten aircraft are divided into three different types of aircraft, each type having a different operating costs in the heterogeneous fleet.

Tables 2 and 3 present the optimal results obtained for a conventional planning model and the proposed planning model considering failure prognostics using a homogeneous fleet, respectively. In both tables the first column specifies the data set tested, where REAL is used for the real instance provided by the fractional operator and GD are the generated instances. The second and third columns respectively present the total deadhead and live flight hours of each schedule. The maintenance costs and total costs are displayed in the

Dataset	Deadhead	Live	Maintenance cost	Total cost	Solver time
Real	102.14	166.54	95,230	793,798	14.57
GD2	130.76	218.25	95,470	1,002,896	20.76
GD3	106.86	224.54	76,240	937,880	18.35
GD4	119.63	235.03	95,470	1,017,586	15.36
GD5	113.2	220.26	95,470	962,466	41.49
GD6	120.69	223.77	62,770	958,366	57.7
GD7	129.01	224.65	95,470	1,014,986	48.36
GD8	108.76	222.93	95,230	957,624	53.98
GD9	116.13	230.92	137,670	1,040,000	35.51
GD10	108.21	217.72	95,470	942,888	51.86
GD11	115.57	238.57	95,230	1,015,890	53.86
GD12	105.84	222.47	80,230	933,836	51.14
GD13	120.07	226.31	137,430	1,038,018	55.24

Table 2. Optimal results using conventional planning for homogeneous fleet

Dataset	Deadhead	Live	Maintenance cost	Total cost	Solver time
Real	105.21	166.54	80,230	786,780	13.76
GD2	131.74	218.25	95,230	1,005,204	18.88
GD3	103	224.54	61,000	912,604	16.73
GD4	126.93	235.03	95,230	1,036,326	15.55
GD5	112.24	220.26	95,230	959,730	45.19
GD6	123.77	223.77	63,010	966,614	53.54
GD7	128.18	224.65	137,670	1,055,028	48.61
GD8	111.15	222.93	95,230	963,838	50.51
GD9	115.06	230.92	137,670	1,037,218	36.2
GD10	105.05	217.72	95,230	934,432	45.06
GD11	115.97	238.57	95,230	1,017,034	51.4
GD12	104.18	222.47	80,230	929,520	49.78
GD13	123.11	226.31	122,430	1,030,922	56.92

Table 3.
Optimal results using proposed model planning for homogeneous fleet

fourth and fifth columns, respectively. The last column shows the total processing time for each instance.

The maintenance and total costs are computed with Equations (15) and (16), by testing the obtained schedules against actual events, including corrective maintenance events. Therefore, maintenance costs include both preventive and corrective maintenance. The solver time is counted in seconds.

The corrective maintenance costs are as follows:

(1) Failure 1 costs:
- In-base: 15,000
- Out-of-base: 30,000

(2) Failure 2 costs:
- In-base: 4,000
- Out-of-base: 8,000

(3) Failure 3 costs:
- In-base: 4,000
- Out-of-base: 8,000

Although the deadhead flight hour tends to be higher for the proposed model considering failure prognostics, this behavior is expected as this model favors legs that provide a better maintenance opportunities. The conventional model on the other hand will always choose the alternative with the lowest deadhead connections. In some cases, however, the model using prognostics data provides a better solution when considering corrective maintenance events as probable failure conditions are taken into consideration.

For the real instance and generated instance GD3, we are able to see this occur, although the deadhead flights are longer, the savings brought on by opportune positioning result in one aircraft being at a maintenance base when it has an unexpected failure that in turn reduces the maintenance costs. Since the reduction in maintenance costs outweighs the extra flight hours, the overall costs are also reduced.

Similarly to Tables 2, 3, 4, and 5 present the optimal results obtained using a heterogeneous fleet for the conventional and proposed planning models, respectively.

Table 4.
Optimal results using conventional model planning for heterogeneous fleet

Dataset	Deadhead	Live	Maintenance cost	Total cost	Solver time
Real	100.19	166.54	93,700	7,874,901	43.83
GD2	129.75	218.25	95,470	10,234,442	107.77
GD3	97.46	224.54	120,210	9,282,613	82.81
GD4	122.83	235.03	95,470	10,509,862	124.47
GD5	117.07	220.26	95,470	9,914,779	95.95
GD6	123.2	223.77	63,010	10,138,987	183.67
GD7	128.92	224.65	95,470	10,390,611	121.03
GD8	115.61	222.93	137,670	10,011,770	88.94
GD9	125.33	230.92	137,670	10,463,869	88.91
GD10	109.71	217.72	95,470	9,689,169	112.73
GD11	119.7	238.57	80,470	10,502,187	95.39
GD12	116.21	222.47	95,470	9,917,416	106.74
GD13	128.94	226.31	137,670	10,467,971	709.62

Table 5.
Optimal results using proposed model planning for heterogeneous fleet

Dataset	Deadhead	Live	Maintenance cost	Total cost	Solver time
Real	99.9	166.54	78,460	7,857,610	43.85
GD2	130.39	218.25	80,470	10,238,194	136.55
GD3	100.55	224.54	78,010	9,295,440	114.63
GD4	126.74	235.03	80,470	10,562,545	92.92
GD5	117.26	220.26	95,470	9,937,633	97.08
GD6	127.29	223.77	63,010	10,255,308	127.76
GD7	129.21	224.65	95,470	10,426,650	86.17
GD8	117.19	222.93	80,470	9,922,047	91.43
GD9	120.03	230.92	80,470	10,262,513	58.93
GD10	110.56	217.72	95,470	9,672,118	127.65
GD11	120.78	238.57	80,470	10,540,863	115.28
GD12	117.43	222.47	80,470	9,935,232	95.23
GD13	125.67	226.31	80,470	10,372,974	152.31

As cited previously, three different aircraft types are considered in this work, and their operating costs are as follows:

(1) Type 1: 2,600

(2) Type 2: 3,000

(3) Type 3: 3,300

In the tested scenarios, the 10 aircraft are divided into 4 of type 1, 3 of type 2 and 3 of type 3.

The planning window used of 20 flights provides an average planning window of three days. In this way, the processing times for all 117 and 119 flights, for real and generated data respectively, in the span of 17 days were within an acceptable limit. Taking into consideration that planning has to be possible under a few hours since new flights may arise with a few hours of notice. Since the processing times for both models are very similar, and for the presented cases are all under 12 min, the decision-maker could compare both models and choose which one better serves their current situation. Although having a heterogeneous fleet weighs a bit more on the solving algorithm when compared to the homogeneous fleet cases, the running times are well within acceptable boundaries.

Tables 6 and 7 present the percentage gains and losses in deadhead flight times and costs provided by the model using prognostics data. In terms of proportional gains, the

Table 6. Comparison for homogeneous fleet

Dataset	ΔDeadhead	ΔTotal cost	ΔMaintenance cost
Real	3.01%	−0.88%	−15.75%
GD2	0.75%	0.23%	−0.25%
GD3	−3.61%	−2.70%	−19.99%
GD4	6.10%	1.84%	−0.25%
GD5	−0.85%	−0.28%	−0.25%
GD6	2.55%	0.86%	0.38%
GD7	−0.64%	3.95%	44.20%
GD8	2.20%	0.65%	0.00%
GD9	−0.92%	−0.27%	0.00%
GD10	−2.92%	−0.90%	−0.25%
GD11	0.35%	0.11%	0.00%
GD12	−1.57%	−0.46%	0.00%
GD13	2.53%	−0.68%	−10.91%
	0.54%	0.11%	−0.24%

Table 7. Comparison for heterogeneous fleet

Dataset	ΔDeadhead	ΔTotal cost	ΔMaintenance cost
Real	−0.29%	−0.22%	−16.26%
GD2	0.49%	0.04%	−15.71%
GD3	3.17%	0.14%	−35.11%
GD4	3.18%	0.50%	−15.71%
GD5	0.16%	0.23%	0.00%
GD6	3.32%	1.15%	0.00%
GD7	0.22%	0.35%	0.00%
GD8	1.37%	−0.90%	−41.55%
GD9	−4.23%	−1.92%	−41.55%
GD10	0.77%	−0.18%	0.00%
GD11	0.90%	0.37%	0.00%
GD12	1.05%	0.18%	−15.71%
GD13	−2.54%	−0.91%	−41.55%
	0.58%	−0.09%	−17.17%

proposed model provides a cost reduction of up to 2.70% and a possible increase in costs of 3.95% for the homogeneous fleet given that there are opportune flights to improve maintenance planning. However, on average, the total cost and deadhead flight hours were 0.11 and 0.54% higher for the proposed model. More deadhead hours are expected since the proposed model focuses on finding better maintenance opportunities rather than fastest route, and in that sense Table 6 shows that on average maintenance costs are reduced by 0.24% for homogeneous fleet. The unexpected average increase in total cost happened because of generated case 7, where although the route encountered by the proposed model was shorter, the preventive maintenance costs incurred were higher than that of the conventional model. The corrective maintenance costs, however, were the same for both models.

Similarly, Tables 6 and 7 provide a comparison between the proposed and conventional routing models for heterogeneous fleet. Like the scenarios using homogeneous fleet, heterogeneous fleet scenarios have longer connecting flight hours and a reduced average maintenance cost. In these cases, the gains in maintenance costs are more significant than the previous scenarios, reflecting on the average total cost reduction.

In the experiments, four different failure events are considered. Tables 8 and 9 show the type of failure that occurred in each event and whether the maintenance was done in or out-of-base in each instance for homogeneous and heterogeneous fleets respectively. The first and second lines enumerate each event and detail what type of failure occurred, respectively. The following lines describe whether each event was done in or out of base for each solution. From Table 8, we see that the proposed model using prognostics data provided four opportunities for in-base maintenance while conventional planning provided only two for homogeneous fleet, reducing maintenance costs. Likewise, for heterogeneous fleet, Table 9 shows that in nine cases the proposed model was able to find routes with in-base maintenance opportunities versus only two in the conventional model.

The benefits of the new plan extend from the operational decision-maker, responsible for determining routes, to the maintenance planner, responsible for scheduling maintenance operations. Apart from the potential cost reductions derived from the new planning, by indicating routes with a greater probability of having aircraft in a maintenance base, the strain on maintenance personnel is reduced. The reduction of out-of-base maintenance events reduces in turn disruptions in the already planned maintenance scheduling.

Even if there are no available routes ending or beginning at maintenance bases, failure prognostics information can assist AOG anticipation and preventive parts, personnel and other equipment allocation. By knowing the failures that might occur and reducing troubleshooting time, as mentioned in Rodrigues *et al.* (2012), the appropriate mechanic can be prepared beforehand.

	Events			
	1	2	3	4
Failure	1	2	3	1
Real	Out	Out	Out	Out
Real FP	In	Out	Out	Out
GD2	Out	Out	Out	Out
GD2 FP	Out	Out	Out	Out
GD3	Out	Out	Out	Out
GD3 FP	In	Out	Out	Out
GD4	Out	Out	Out	Out
GD4 FP	Out	Out	Out	Out
GD5	Out	Out	Out	Out
GD5 FP	Out	Out	Out	Out
GD6	Out	Out	Out	In
GD6 FP	Out	Out	Out	In
GD7	Out	Out	Out	Out
GD7 FP	Out	Out	Out	Out
GD8	Out	Out	Out	Out
GD8 FP	Out	Out	Out	Out
GD9	Out	Out	Out	Out
GD9 FP	Out	Out	Out	Out
GD10	Out	Out	Out	Out
GD10 FP	Out	Out	Out	Out
GD11	Out	Out	Out	Out
GD11 FP	Out	Out	Out	Out
GD12	Out	Out	Out	In
GD12 FP	Out	Out	Out	In
GD13	Out	Out	Out	Out
GD13 FP	Out	Out	Out	In

Table 8.
Corrective maintenance events for homogeneous fleet

	Events			
	1	2	3	4
Failure	1	2	3	1
Real	Out	Out	Out	Out
Real FP	In	Out	Out	Out
GD2	Out	Out	Out	Out
GD2 FP	In	Out	Out	Out
GD3	Out	Out	Out	Out
GD3 FP	Out	Out	Out	Out
GD4	Out	Out	Out	Out
GD4 FP	Out	Out	Out	In
GD5	Out	Out	Out	Out
GD5 FP	Out	Out	Out	Out
GD6	Out	Out	Out	In
GD6 FP	Out	Out	Out	In
GD7	Out	Out	Out	Out
GD7 FP	Out	Out	Out	Out
GD8	Out	Out	Out	Out
GD8 FP	Out	Out	Out	In
GD9	Out	Out	Out	Out
GD9 FP	Out	Out	Out	In
GD10	Out	Out	Out	Out
GD10 FP	Out	Out	Out	Out
GD11	Out	Out	Out	In
GD11 FP	Out	Out	Out	In
GD12	Out	Out	Out	Out
GD12 FP	Out	Out	Out	In
GD13	Out	Out	Out	Out
GD13 FP	Out	Out	Out	In

Table 9. Corrective maintenance events for heterogeneous fleet

5. Conclusion

The present work presents a novel approach to optimize the routing problem of fractional fleets. A mixed integer formulation was used for both baseline planning without failure prognostics and the proposed model, which considers failure prognostics in planning. Exact solutions were obtained using the Gurobi solver previously mentioned.

Novel considerations presented in this work include more flexible preventive maintenance planning with larger windows depending on aircraft activity and the inclusion of failure prognostics information from data-driven algorithms based on previous works.

Within the context of the problem, this work contributes to the exploration of an important and promising part of the problem as a whole. The modeling, even with the premises observed, addresses the supportability of homogeneous and heterogeneous fleets of executive aircraft in a fractional fleet when scheduling flights taking into account both legacy standards of preventive maintenance and the use of information from prognostic models. Naturally, this work paves the way for other modeling by eliminating the premises and completing the systemic understanding of the aircraft allocation problem.

Since the nature of this problem is inherently dynamic due to the operating conditions of fractional airlines, an agile planning strategy is essential for efficient operation. For this reason, the processing time of the model needed to remain within a few hours for adaptation to practical use. Object that was accomplished seeing as the processing times of all instances remained under 12 min.

As can be seen from the presented results, the reduction in maintenance costs may be greater than the added flight hours depending on the flight hour costs and maintenance costs. Even when maintenance opportunities are not found, total costs do not increase significantly due to the increased flight hours. Given that, business aviation is already a costly and highly competitive sector, any improvement in operations may have a significant impact.

The better awareness of possible maintenance events can provide a more efficient planning opportunity for the maintenance planner, making better use of available man power. By having more maintenance events happening in-base, rather than out-of-base, the original planning will suffer less disruptions.

Future works include the inclusion of anticipating corrective maintenance activities to prevent AOG events at the model and the usage of stochastic modeling and optimization solutions to properly estimate and minimize operational risks yielded in the form of situation probabilities and their impacts.

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